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Remarks on the Hilfer fractional derivative

Abstract. The purpose of the paper is to familiarise the reader with the concept of the Hilfer fractional derivative. The definition and basic properties of the Hilfer derivative are given. Formulas for the derivatives of selected functions are derived and compared with the corresponding Riemann–Liouville and Caputo derivatives. Examples of calculating the derivatives of basic functions are presented. The paper also contains a number of self-solving exercises, with answers.
Keywords: fractional derivative, Hilfer derivative, Caputo derivative, Riemann–Liouville derivative, fractional calculus.

1. Introduction

Fractional calculus is a rapidly growing branch of mathematics in recent decades. Its historical development — from Leibniz's 1695 correspondence with de l'Hospital, through the work of Liouville and Riemann, to the emergence of the Riemann–Liouville and Grünwald–Letnikov derivatives — as well as the list of conditions that any reasonable fractional-order derivative is expected to satisfy, namely linearity, the identity property at $\alpha = 0$, consistency with classical differentiation at integer orders, the index law, and a generalized Leibniz rule, have been discussed in detail in the paper [15] devoted to the Caputo derivative (see also [2, 6–13]). They are not repeated here, and only the facts indispensable for introducing the operator studied in the present paper are recalled.

For $\alpha \in [n-1, n)$, $n \in \mathbb{N}$, the Riemann–Liouville derivative is obtained by first applying a fractional integral to f and then taking the nth derivative, $D^\alpha f(t) = D^n I^{n-\alpha} f(t)$, where $D^n f(t) = f^{(n)}(t)$ denotes the classical derivative of order n. The Caputo derivative, introduced by Michele Caputo in 1967 in connection with boundary value problems arising in the theory of viscoelasticity [1], reverses the order of these two operations, ${}^C D^\alpha f(t) = I^{n-\alpha} D^n f(t)$. Both operators satisfy the conditions recalled above, but they differ in an important practical respect. The Riemann–Liouville derivative of a nonzero constant does not vanish, and hence the initial conditions in a Cauchy problem formulated in terms of this operator do not have the usual physical interpretation in terms of $f(0), f'(0), \ldots$. The Caputo derivative avoids this difficulty, which is one of the main reasons for its widespread use in applications.

In 2000, Rudolf Hilfer proposed an operator that contains both of the above derivatives as special cases [4]. Besides the order α, the Hilfer derivative depends on an additional type parameter $\beta \in [0, 1]$, which

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places the classical derivative D^n between two fractional integrations whose orders are governed jointly by α and β. For $\beta = 0$ the resulting operator coincides with the Riemann–Liouville derivative, for $\beta = 1$ with the Caputo derivative, and for $0 < \beta < 1$ it defines a genuinely new, intermediate operator. This extra degree of freedom has attracted considerable attention in the recent fractional-calculus literature, both from the theoretical point of view [3,5] and in applications to physical models [4,14].

In this paper, the Hilfer fractional order derivative is studied. The definition and basic properties of the operator are presented and proved, its values for a few elementary functions are derived and compared with the Riemann–Liouville and Caputo values, and a number of worked and self-solving examples are given.

2. Definition of the Hilfer fractional order derivative

Let f be a real-valued function integrable on every closed interval $[0,t]$, $t > 0$, i.e. $f \in L^1([0,t],\mathbb{R})$. As in the classical Riemann–Liouville and Caputo theory, the starting point is the Riemann–Liouville fractional integral.

Definition 1. *If $f \in L^1([0,t],\mathbb{R})$, $t \in \mathbb{R}_+$, and $\alpha \in \mathbb{R}_+$, then*

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau \tag{1}$$

is called the Riemann–Liouville fractional integral of order α, and $I^0 f := f$.

Two classical fractional derivatives can be built from the pair (I^α, D^n), $n = \lceil \alpha \rceil$, according to the order in which the two operations are carried out: differentiating first and integrating afterwards yields the Riemann–Liouville derivative ($D^\alpha f(t) = D^n I^{n-\alpha} f(t)$), while integrating first and differentiating afterwards yields the Caputo derivative ($^C D^\alpha f(t) = I^{n-\alpha} D^n f(t)$). Hilfer's construction [4] treats these two orderings as the extreme members of a one-parameter family, obtained by splitting the "missing" fractional order $n - \alpha$ between an integration performed before D^n and one performed after it.

Definition 2. *Fix $n-1 < \alpha < n$, $n \in \mathbb{N}$, and a type $\beta \in [0,1]$. The left-sided Hilfer fractional derivative of order α and type β of f is the operator*

$$D^{\alpha,\beta} f(t) = I^{\beta(n-\alpha)} D^n I^{(1-\beta)(n-\alpha)} f(t), \tag{2}$$

provided that $I^{(1-\beta)(n-\alpha)} f$ is n times differentiable and the outer integral $I^{\beta(n-\alpha)}$ of the resulting function exists.

Setting $\beta = 1$ places the entire missing order $n-\alpha$ after D^n, which is exactly the Caputo construction, $D^{\alpha,1} f(t) = {}^C D^\alpha f(t)$; setting $\beta = 0$ places it entirely before D^n, recovering $D^{\alpha,0} f(t) = D^\alpha f(t)$. Values of β strictly between 0 and 1 divide the order $n-\alpha$ between the two integrations and produce operators that coincide with neither classical derivative. Being built from fractional integrals, $D^{\alpha,\beta}$ retains the non-local, memory-type character of D^α and ${}^C D^\alpha$: the value $D^{\alpha,\beta} f(t)$ depends on the whole restriction of f to $[0,t]$, not on a neighbourhood of t alone.

All properties and examples discussed below concern the branch $0 < \alpha < 1$ ($n = 1$), for which (2) becomes

$$D^{\alpha,\beta} f(t) = I^{\beta(1-\alpha)} D^1 I^{(1-\beta)(1-\alpha)} f(t), \qquad D^{\alpha,0} f(t) = D^\alpha f(t), \quad D^{\alpha,1} f(t) = {}^C D^\alpha f(t). \tag{3}$$

3. Properties of the Hilfer fractional derivative

Throughout this section $f, g\colon \mathbb{R}_+ \to \mathbb{R}$ are such that all the Hilfer derivatives considered exist. The five statements below establish, in turn, the identity property, consistency with integer-order differentiation, linearity, the index law and a generalized Leibniz rule for $D^{\alpha,\beta}$ — the same list of requirements mentioned in the Introduction. Only the identity property turns out to depend qualitatively on whether β equals 1 or not.

Theorem 1 (Identity property). *If $0 \leqslant \beta < 1$, then $\lim\limits_{\alpha\to 0^+} D^{\alpha,\beta} f(t) = f(t)$ for every f continuous near t. If $\beta = 1$, then $\lim\limits_{\alpha\to 0^+} D^{\alpha,1} f(t) = f(t) - f(0)$, and, exactly as for the Caputo derivative, the value $D^{0,1} f := f$ is adopted by convention.*

Proof. Set $\nu = (1-\beta)(1-\alpha)$ in (3). Then

$$D^{\alpha,\beta} f(t) = I^{\beta(1-\alpha)} D^1 I^\nu f(t).$$

As $\alpha \to 0^+$, we have

$$\nu \to 1-\beta, \qquad \beta(1-\alpha) \to \beta.$$

Hence

$$\lim_{\alpha\to 0^+} D^{\alpha,\beta} f(t) = I^\beta D^1 I^{1-\beta} f(t) = I^\beta D^\beta f(t),$$

where $D^\beta f(t) := D^1 I^{1-\beta} f(t)$ denotes the Riemann–Liouville derivative of order β.

It is known [10] that

$$I^\beta D^\beta f(t) = f(t) - \frac{t^{\beta-1}}{\Gamma(\beta)}\, c, \qquad c := \lim_{s\to 0^+} I^{1-\beta} f(s).$$

For $0 < \beta < 1$, the constant c vanishes. Indeed, choose $t_0 > 0$ such that f is continuous on $[0, t_0]$, and set

$$\gamma = 1 - \beta \in (0,1), \qquad M = \sup_{[0,t_0]} |f|.$$

Then, for $0 < s \leqslant t_0$,

$$\left| I^\gamma f(s) \right| \leqslant \frac{M}{\Gamma(\gamma)} \int_0^s (s-\tau)^{\gamma-1}\, d\tau = \frac{M}{\Gamma(\gamma+1)}\, s^\gamma \xrightarrow[s\to 0^+]{} 0.$$

Thus $c = 0$ and consequently

$$I^\beta D^\beta f(t) = f(t).$$

For $\beta = 0$, the prefactor $\frac{1}{\Gamma(\beta)}$ vanishes, so the same conclusion follows immediately. For $\beta = 1$, however,

$$c = I^0 f(0^+) = f(0),$$

which need not be zero. In this case,

$$I^1 D^1 f(t) = \int_0^t f'(\tau)\, d\tau = f(t) - f(0),$$

which gives the Caputo-type behaviour stated above. □

Theorem 2 (Consistency with integer-order differentiation). *For every $n \in \mathbb{N}$ and every $\beta \in [0,1]$, $\lim_{\alpha \to n^-} D^{\alpha,\beta} f(t) = D^n f(t)$, regardless of β.*

Proof. As $\alpha \to n^-$, both exponents in (2) tend to 0, whatever the value of $\beta \in [0,1]$, and since I^0 is the identity operator, $D^{\alpha,\beta} f(t) \to D^n f(t)$. □

Theorem 3 (Linearity). *For every $a, b \in \mathbb{R}$, $D^{\alpha,\beta}\big(af + bg\big)(t) = a\, D^{\alpha,\beta} f(t) + b\, D^{\alpha,\beta} g(t)$.*

Proof. Each of the three operators composed in (2) — the integrals $I^{\beta(n-\alpha)}$, $I^{(1-\beta)(n-\alpha)}$ and the classical derivative D^n — is linear, and a composition of linear operators is linear. □

Theorem 4 (Index law). *Under suitable smoothness conditions on f, analogous to those required for the Riemann–Liouville and Caputo derivatives, and for $\alpha, \gamma \in \mathbb{R}_+$, $\beta, \delta \in [0,1]$,*

$$D^{\alpha,\beta}\ D^{\gamma,\delta} f(t) = D^{\alpha+\gamma,*} f(t)$$

holds for a type $*$ determined by $\alpha, \beta, \gamma, \delta$.

Theorem 5 (Generalized Leibniz rule). *$D^{\alpha,\beta}$ satisfies a generalized Leibniz rule for the fractional derivative of a product, analogous to those known for D^α and ${}^C D^\alpha$.*

Neither Theorem 4 nor Theorem 5 holds unconditionally, in the same way that the corresponding statements fail for arbitrary f in the Riemann–Liouville and Caputo theory. Both are proved by writing $D^{\alpha,\beta}$ through Riemann–Liouville operators, a route not pursued here — see [4, 5, 10].

Theorems 1–5 together show that $D^{\alpha,\beta}$ meets the five requirements recalled in the Introduction, subject to the same reservation noted for the Caputo derivative at $\beta = 1$: the identity property there holds only after the convention $D^{0,1} f := f$ is adopted.

4. The Hilfer fractional derivatives of selected functions

We now compute $D^{\alpha,\beta}$ for three elementary functions, restricting attention, as before, to $0 < \alpha < 1$.

Constant function

$$f(t) = c,\ c \in \mathbb{R}$$

Theorem 6. *For $c \in \mathbb{R}$,*

$$D^{\alpha,\beta}c = \begin{cases} \dfrac{c}{\Gamma(1-\alpha)}\, t^{-\alpha}, & 0 \leqslant \beta < 1, \\ 0, & \beta = 1. \end{cases} \tag{4}$$

Proof. Set $\nu = (1-\beta)(1-\alpha)$. For $\beta < 1$ we have $\nu > 0$, so

$$I^{\nu} c = \frac{c}{\Gamma(\nu+1)}\, t^{\nu} \quad \Longrightarrow \quad \frac{d}{dt} I^{\nu} c = \frac{c}{\Gamma(\nu)}\, t^{\nu-1}.$$

Applying $I^{\beta(1-\alpha)}$ by means of $I^{q} t^{r} = \frac{\Gamma(r+1)}{\Gamma(r+q+1)} t^{r+q}$, with $q = \beta(1-\alpha)$, $r = \nu - 1$, gives $r + 1 = \nu$ and, since $\nu + \beta(1-\alpha) = 1 - \alpha$,

$$r + q = \nu - 1 + \beta(1-\alpha) = -\alpha.$$

The factor $\Gamma(\nu)$ therefore cancels, leaving

$$D^{\alpha,\beta}c = \frac{c}{\Gamma(1-\alpha)}\, t^{-\alpha},$$

a value free of β, valid for every $\beta \in [0,1)$. For $\beta = 1$ we have $\nu = 0$, so $I^{0} c = c$ is already constant, its ordinary derivative vanishes, and $I^{1-\alpha} 0 = 0$. Hence $D^{\alpha,\beta}c = 0$. □

For $\beta < 1$ the value obtained is exactly the Riemann–Liouville derivative $D^{\alpha}c$, which is generally nonzero; only at $\beta = 1$ does it drop to the Caputo value ${}^{C}D^{\alpha}c = 0$ (see [15]).

Example 1. Take $c = 7$ and $\alpha = \frac{2}{5}$. For $\beta < 1$,

$$D^{\frac{2}{5},\beta}\, 7 = \frac{7}{\Gamma(\frac{3}{5})}\, t^{-\frac{2}{5}},$$

while $D^{\frac{2}{5},1}\, 7 = 0$.

Power function

$$f(t) = t^{p},\ p > 0$$

Theorem 7. *For $p > 0$, and independently of $\beta \in [0,1]$,*

$$D^{\alpha,\beta} t^{p} = \frac{\Gamma(p+1)}{\Gamma(p-\alpha+1)}\, t^{p-\alpha}. \tag{5}$$

Proof. With $\nu = (1-\beta)(1-\alpha)$ as above and $p > 0$,

$$I^{\nu} t^{p} = \frac{\Gamma(p+1)}{\Gamma(p+\nu+1)}\, t^{p+\nu} \quad \Longrightarrow \quad D^{1} I^{\nu} t^{p} = \frac{\Gamma(p+1)}{\Gamma(p+\nu)}\, t^{p+\nu-1}.$$

Applying $I^{\beta(1-\alpha)}$ with $q = \beta(1-\alpha)$, $r = p+\nu-1$ gives $r+1 = p+\nu$ and, again since $\nu+\beta(1-\alpha) = 1-\alpha$,

$$r + q = p + \nu - 1 + \beta(1-\alpha) = p - \alpha.$$

Hence

$$D^{\alpha,\beta}t^p = \frac{\Gamma(p+1)}{\Gamma(p+\nu)} \cdot \frac{\Gamma(p+\nu)}{\Gamma(p-\alpha+1)}\, t^{p-\alpha} = \frac{\Gamma(p+1)}{\Gamma(p-\alpha+1)}\, t^{p-\alpha},$$

with ν – and hence β – cancelling out entirely. □

This is precisely the common value already known for the Riemann–Liouville and Caputo derivatives, $D^\alpha t^p = {}^C D^\alpha t^p = \frac{\Gamma(p+1)}{\Gamma(p-\alpha+1)}\, t^{p-\alpha}$. In contrast with the constant function above, once the exponent is strictly positive the two classical derivatives already agree, so the type β that interpolates between them has nothing left to do.

Example 2. Take $p = 3$ and $\alpha = \frac{3}{5}$. For every $\beta \in [0,1]$,

$$D^{\frac{3}{5},\beta}t^3 = \frac{\Gamma(4)}{\Gamma(\frac{17}{5})}\, t^{\frac{12}{5}} = \frac{6}{\Gamma(\frac{17}{5})}\, t^{\frac{12}{5}}.$$

Exponential function

$$f(t) = e^{at},\ a \in \mathbb{R}$$

Theorem 8. *For $a \in \mathbb{R}$,*

$$D^{\alpha,\beta}e^{at} = \begin{cases} t^{-\alpha}E_{1,1-\alpha}(at), & 0 \leqslant \beta < 1, \\ a\, t^{1-\alpha}E_{1,2-\alpha}(at), & \beta = 1, \end{cases} \tag{6}$$

where

$$E_{\mu,\lambda}(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma(\mu k + \lambda)}$$

is the two-parameter Mittag-Leffler function.

Proof. Insert the power series $e^{at} = \sum_{k=0}^{+\infty} \frac{a^k}{k!}t^k$ into $D^{\alpha,\beta}$ and apply Theorem 3 termwise. For $k \geqslant 1$ the term $\frac{a^k}{k!}t^k$ is governed by the power-function formula (5), contributing $\frac{a^k}{\Gamma(k-\alpha+1)}t^{k-\alpha}$ irrespective of β; the $k = 0$ term is the constant 1, governed instead by (4). For $\beta < 1$ every term, including $k = 0$, matches the pattern $\frac{a^k}{\Gamma(k+1-\alpha)}t^{k-\alpha}$, so summing over $k \geqslant 0$,

$$D^{\alpha,\beta}e^{at} = t^{-\alpha}\sum_{k=0}^{+\infty} \frac{(at)^k}{\Gamma(k+1-\alpha)} = t^{-\alpha}E_{1,1-\alpha}(at).$$

For $\beta = 1$ the $k = 0$ term drops out (it equals 0 rather than $\frac{t^{-\alpha}}{\Gamma(1-\alpha)}$), so

$$D^{\alpha,1}e^{at} = t^{-\alpha}\Big[E_{1,1-\alpha}(at) - \tfrac{1}{\Gamma(1-\alpha)}\Big].$$

The recursion $E_{1,\gamma}(z) = zE_{1,\gamma+1}(z) + \frac{1}{\Gamma(\gamma)}$, applied with $z = at$, $\gamma = 1-\alpha$, turns the bracket into $at\, E_{1,2-\alpha}(at)$, giving $D^{\alpha,1}e^{at} = a\, t^{1-\alpha}E_{1,2-\alpha}(at)$. □

The upper branch, valid for $\beta < 1$, is the Riemann–Liouville derivative $D^\alpha e^{at}$; the lower branch, at $\beta = 1$, is the Caputo derivative ${}^C D^\alpha e^{at}$ presented in [15].

Example 3. For $\alpha = \frac{1}{3}$ and $a = 2$, by (6),

$$D^{\frac{1}{3},\beta} e^{2t} = t^{-\frac{1}{3}} E_{1,\frac{2}{3}}(2t) \quad (\beta < 1), \qquad D^{\frac{1}{3},1} e^{2t} = 2\, t^{\frac{2}{3}} E_{1,\frac{5}{3}}(2t).$$

Theorems 6 and 7 exhibit, on specific functions, the same dichotomy already met in the identity property: whenever $f(0) \neq 0$, the value $D^{\alpha,\beta} f(t)$ is completely insensitive to β throughout $[0, 1)$ and jumps only at the single point $\beta = 1$. The type parameter should therefore not be pictured as continuously blending Riemann–Liouville and Caputo values. Its role is rather to select which of the two associated families of initial conditions is attached to the resulting fractional differential equation.

5. Exercises

To let the reader practise working with the two-parameter operator $D^{\alpha,\beta}$, a short set of exercises is collected below, together with the corresponding answers.

Exercise 1. Calculate the Hilfer derivative of order $\alpha = \frac{1}{2}$ and type $\beta = \frac{1}{3}$ of the constant function $f(t) = 6$.

Answer: $D^{\frac{1}{2},\frac{1}{3}}\, 6 = \dfrac{6}{\sqrt{\pi t}}$.

Exercise 2. Calculate the Hilfer derivative of order $\alpha = \frac{1}{2}$ and type $\beta = 1$ of the constant function $f(t) = 6$.

Answer: $D^{\frac{1}{2},1}\, 6 = 0$.

Exercise 3. Calculate the Hilfer derivative of order $\alpha = \frac{3}{4}$ of the function $f(t) = t$, for an arbitrary type $\beta \in [0, 1]$.

Answer: $D^{\frac{3}{4},\beta}\, t = \dfrac{1}{\Gamma(\frac{5}{4})} t^{\frac{1}{4}}$, independently of β.

Exercise 4. Calculate the Hilfer derivative of order $\alpha = \frac{1}{3}$ of the function $f(t) = t^2 + 5$, for $\beta = 0$ and for $\beta = 1$, and compare the two results.

Answer: $D^{\frac{1}{3},0}(t^2 + 5) = \dfrac{2}{\Gamma(\frac{8}{3})} t^{\frac{5}{3}} + \dfrac{5}{\Gamma(\frac{2}{3})} t^{-\frac{1}{3}}$; $\quad D^{\frac{1}{3},1}(t^2 + 5) = \dfrac{2}{\Gamma(\frac{8}{3})} t^{\frac{5}{3}}$.

Exercise 5. Calculate the Hilfer derivative of order $\alpha = \frac{1}{2}$ of the function $f(t) = e^{2t}$, for $\beta = 0$ and for $\beta = 1$.

Answer: $D^{\frac{1}{2},0} e^{2t} = t^{-\frac{1}{2}} E_{1,\frac{1}{2}}(2t)$; $\quad D^{\frac{1}{2},1} e^{2t} = \sqrt{2}\, e^{2t} \operatorname{erf}(\sqrt{2t})$.

6. Final remarks

The Hilfer fractional derivative provides a common, two-parameter framework in which the Riemann–Liouville and Caputo fractional derivatives are recovered as the two extreme cases $\beta = 0$ and $\beta = 1$, respectively. As shown in this paper, the type parameter β does not, in general, interpolate the numerical value of the derivative continuously. Instead it selects, in a somewhat discontinuous way, between a Riemann–Liouville-type and a Caputo-type behaviour, with the latter appearing only at the single value $\beta = 1$. This makes the Hilfer derivative particularly useful in the formulation of initial-value problems that need not be restricted to purely Riemann–Liouville or purely Caputo initial conditions, and it has motivated a growing number of applications in physics and in the theory of fractional differential equations [3, 4, 14]. As with the Caputo derivative discussed in [15], mathematical models based on the Hilfer derivative allow the modeller an additional degree of freedom — the type β — which can be tuned, alongside the order α, to fit experimental data more accurately than integer-order or single-parameter fractional-order models.

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